Eur. Phys. J. C 50, 35–46 (2007)
DOI 10.1140/epjc/s10052-006-0188-0

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article – Theoretical Physics

Supersymmetric contributions to $B \to DK$ and the determination of angle γ

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Received: 13 August 2006 /
Published online: 8 February 2007 –

Abstract. We analyze supersymmetric contributions to the branching ratios and CP asymmetries of $B^- \to D^0 K^-$ and $B^- \to \bar{D}^0 K^-$ processes. We investigate the possibility that supersymmetric CP violating phases can affect our determination for the angle γ in the unitary triangle of Cabibbo–Kobayashi–Maskawa mixing matrix. We calculate the gluino and chargino contributions to $b \to u(\bar{c}s)$ and $b \to c(\bar{u}s)$ transitions in a model independent way by using the mass insertion approximation method. We also revise the D^0–$\bar{D}^0$ mixing constraints on the mass insertions between the first and second generations of the up sector. We emphasize that in case of negligible D^0–$\bar{D}^0$ mixing, one should consider simultaneous contributions from more than one mass insertion in order to be able to obtain the CP asymmetries of these processes within their 1σ experimental range. However, with a large D^0–$\bar{D}^0$ mixing, one finds a significant deviation between the two asymmetries and it becomes natural to have them of the order of the central values of their experimental measurements.

1 Introduction

Recently, the BaBar collaborations have measured the charge CP asymmetries $A_{CP_\pm}$ and the branching ratios $R_{CP_\pm}$ of the $B^- \to D^0 K^-$ and $B^- \to \bar{D}^0 K^-$ decays [1]. The following results have been reported:

$$\begin{aligned} A_{CP_+} &= 0.35 \pm 0.13\,(\text{stat}) \pm 0.04\,(\text{syst})\,, \\ A_{CP_-} &= -0.06 \pm 0.13\,(\text{stat}) \pm 0.04\,(\text{syst})\,, \end{aligned} \quad (1)$$

$$\begin{aligned} R_{CP_+} &= 0.90 \pm 0.12\,(\text{stat}) \pm 0.04\,(\text{syst})\,, \\ R_{CP_-} &= 0.86 \pm 0.10\,(\text{stat}) \pm 0.05\,(\text{syst})\,. \end{aligned} \quad (2)$$

These results, with all other B-factories measurements, provide a stringent test of the standard model (SM) picture of flavor structure and CP violation and open the possibility of probing virtual effects from new physics at low energy.

In the SM, CP violation arises from complex Yukawa couplings which lead to the angles α, β and γ in the unitary triangle of the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix. The angle $\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$ has been determined by the CP asymmetry in the $B^0 \to J/\psi K_S$ process, which is dominated by a tree level contribution. Concerning the angle $\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$, it is believed that a theoretically clean measurement of this angle can be obtained from exploiting the interference between $B^- \to D^0 K^-$ and $B^- \to \bar{D}^0 K^-$, when D^0 and $\bar{D}^0$ mesons decay to the same CP eigenstate [2–4].

At the quark level, the $B^- \to \bar{D}^0 K^-$ and $B^- \to D^0 K^-$ decays are based on the $b \to u(\bar{c}s)$ and $b \to c(\bar{u}s)$ transitions respectively. Therefore, their SM contributions at tree level are suppressed by the CKM factors $V_{cs}V_{ub}^*$ and $V_{us}^*V_{cb}$, which are of order 10^{-3}. This gives the hope that new physics beyond the SM may be possible, like supersymmetry, which contributes to these decays at one loop level, manifesting itself, and bringing about competition with the SM. In this paper we aim to investigate this possibility and check, in a model independent way, whether supersymmetry can significantly modify the CP asymmetries in $B^- \to DK^-$ processes and hence affects the determination of the angle γ. Therefore, we perform a systematic analysis of the SUSY contributions to the $B \to DK$ processes. We compute SUSY contributions to the $b \to u(\bar{c}s)$ and $b \to c(\bar{u}s)$ transitions through gluino and chargino exchange, using the mass insertion approximation method. This approximation is a quite useful tool for studying the SUSY contributions to the flavor processes in a model independent way. We show that the gluino box diagrams give the dominant SUSY contribution, while the chargino exchanges lead to subdominant contributions.

It turns out that D^0–$\bar{D}^0$ mixing may limit the gluino contribution to $B^- \to DK^-$ due to the stringent constraints on the mass insertions between the first and second generations in the up sector, $(\delta^u_{AB})_{12}$. Thus in our analy-

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sis, we revise the D^0–$\bar{D}^0$ mixing constraints [5] and take them into account. We find that with a single mass insertion, the SUSY contribution to $B^- \to DK^-$ decay will be much smaller than the SM result. Nevertheless, with simultaneous contributions from more than one mass insertion, the SUSY effect can be enhanced, and the results of the CP asymmetries become within 1σ experimental range, while the D^0–$\bar{D}^0$ mixing constraints are satisfied.

The paper is organized as follows. In Sect. 2 we study the CP asymmetries and the branching ratios of $B^- \to DK^-$ in the SM. We show that in the SM the branching ratios $R_{CP_\pm}$ are within the experimental range, while the CP asymmetry A_{CP_+} is below its 1σ experimental lower bound and the value of A_{CP_-} is typically negative. In Sect. 3 we compute the gluino and chargino contributions to the $b \to u(\bar{c}s)$ and $b \to c(\bar{u}s)$ transitions in terms of the mass insertions. Section 4 is devoted to the analysis of the SUSY contribution to D^0–$\bar{D}^0$ mixing and revises the possible constraint on the mass insertions $(\delta^u_{AB})_{12}$. The analysis of the SUSY contribution to the CP asymmetries $A_{CP_\pm}$ and the branching ratios $R_{CP_\pm}$ is given in Sect. 5. We show that, in case of negligible D^0–$\bar{D}^0$ mixing, one should consider simultaneous contributions from more than one mass insertion in order to obtain $A_{CP_\pm}$ within their 1σ experimental range. Nevertheless, the usual relation $A_+ \simeq -A_-$, which is valid in the SM, remains. With large D^0–$\bar{D}^0$ mixing, one finds a significant deviation between A_+ and A_-, and it becomes natural to obtain $A_{CP_\pm}$ of the order of the central values of their experimental measurements. Finally, we give our conclusions in Sect. 6.

2 $B^- \to DK^-$ in the standard model

In this section we analyze the CP violation in $B^- \to DK^-$ decays within the SM. The possible quark level topologies of $B^- \to DK^-$ that contribute to the amplitudes $A(B^- \to D^0K^-)$ and $A(B^- \to \bar{D}^0K^-)$ in the SM can be classified to the following three categories, as shown in Fig. 1: color favored tree (T), color suppressed tree (C) and annihilation (A).

These processes are given in terms of the CKM factors $\lambda_c = V_{cb}V^*_{us}$, $\lambda_u = V_{ub}V^*_{cs}$. The decay $B^- \to D^0K^-$ receives contributions from T and C with the factor λ_c, while $B^- \to \bar{D}^0K^-$ get contributions from C and A in terms of λ_u. Since the contributions from the annihilation process to the matrix elements are quite suppressed at the leading order correction [6, 7], it is quite reasonable to assume that $A = 0$. In our analysis we will adopt this approximation, and therefore the general parametrization of the SM amplitudes of $B^- \to DK^-$ decays can be given by

$$A^{\rm SM}(B^- \to D^0K^-) = |A_1|\mathrm{e}^{\mathrm{i}\delta_1} \equiv \bar{T} + \bar{C}\,, \qquad (3)$$

$$A^{\rm SM}(B^- \to \bar{D}^0K^-) = |A_2|\mathrm{e}^{\mathrm{i}\delta_2}\mathrm{e}^{\mathrm{i}\gamma} \equiv C\,, \qquad (4)$$

where the δ_i, $i = 1, 2$, are the strong (CP-conserving) phases. $\bar{T}$ and $\bar{C}$ refer to the color allowed and color suppressed tree amplitudes involving $b \to c(\bar{u}s)$ while C is related to the process $b \to u(\bar{c}s)$. In terms of the two CP eigenstates of the neutral D meson system, $D^0_{CP_\pm} = (D^0 \pm \bar{D}^0)/\sqrt{2}$, one considers the ratios $R_{CP_\pm}$ of the charged averaged partial rates and the charge asymmetries $A_{CP_\pm}$:

$$R_{CP_\pm} = \frac{2\left[\Gamma\left(B^- \to D^0_{CP_\pm}K^-\right) + \Gamma\left(B^+ \to D^0_{CP_\pm}K^+\right)\right]}{\Gamma(B^- \to D^0K^-) + \Gamma(B^+ \to \bar{D}^0K^+)}, \qquad (5)$$

$$A_{CP_\pm} = \frac{\Gamma\left(B^- \to D^0_{CP_\pm}K^-\right) - \Gamma\left(B^+ \to D^0_{CP_\pm}K^+\right)}{\Gamma\left(B^- \to D^0_{CP_\pm}K^-\right) + \Gamma\left(B^+ \to D^0_{CP_\pm}K^+\right)}\,. \qquad (6)$$

We define the ratio of the SM amplitudes of $B^- \to \bar{D}^0K^-$ and $B^- \to D^0K^-$ as

$$r_B\mathrm{e}^{\mathrm{i}\delta_B}\mathrm{e}^{\mathrm{i}\gamma} = \frac{A^{\rm SM}(B^- \to \bar{D}^0K^-)}{A^{\rm SM}(B^- \to D^0K^-)}\,. \qquad (7)$$

According to (3) and (4), $r_B = |A_2/A_1|$ and $\delta_B = \delta_2 - \delta_1$. Using this parametrization, one finds that $R_\pm \equiv R_{CP_\pm}$ is

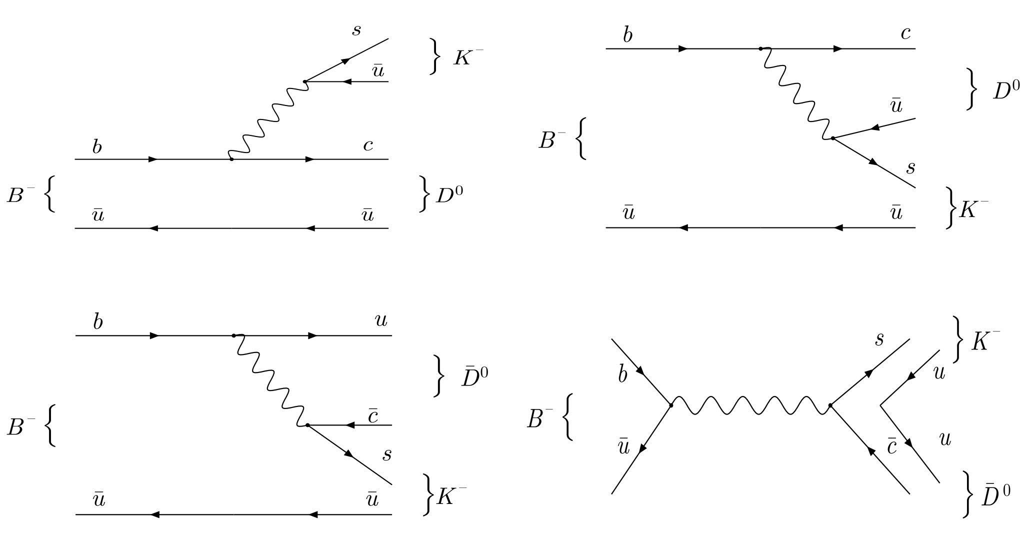

Fig. 1. SM contributions to $B^- \to DK^-$: color-favored tree (*left up*), color-suppressed tree (*right-up* and *left-down*) and annihilation (*right-down*)

given by

$$R_{\pm} = 1 + r_B^2 \pm 2 r_B \cos\delta_B \cos\gamma \,, \tag{8}$$

and $A_{\pm} \equiv A_{CP_{\pm}}$ takes the form

$$A_{\pm} = \frac{\pm 2 r_B \sin\delta_B \sin\gamma}{1 + r_B^2 \pm 2 r_B \cos\delta_B \cos\gamma} \equiv \frac{\pm 2 r_B \sin\delta_B \sin\gamma}{R_{CP_{\pm}}} \,. \tag{9}$$

From (8) one gets

$$\cos\gamma = \frac{R_+ - R_-}{4 r_B \cos\delta_B} \,. \tag{10}$$

Thus, by using the expressions for the CP asymmetries $A_{\pm}$ in (9), one can factorize the dependence on the strong phase, and one gets the following expression for the angle γ in terms of $R_{\pm}$, $A_{\pm}$ and r_B only:

$$\sin\gamma = \frac{2\cos\gamma\,(A_+ - A_-)}{\sqrt{16 r_B^2 \cos^2\gamma - (R_+ - R_-)^2}} \frac{R_+ R_-}{R_+ + R_-} \,. \tag{11}$$

From this expression, one can easily see that the central experimental values of $R_{\pm}$ and $A_{\pm}$ with $r_B \simeq 0.1$ imply that the angle γ is of order $\gamma \simeq 71°$. It is worth mentioning that within the SM, the effect of the D^0–$\bar{D}^0$ mixing on extracting the angle γ using the $B^- \to DK^-$ decays is very small. As emphasized in [8], neglecting this mixing implies an error in determining γ of order 0.1–1°.

In order to analyze the SM predictions for the $A_{\pm}$ and $R_{\pm}$ and compare them with the experimental results reported in (1) and (2), let us consider the SM contributions to the $b \to u(\bar{c}s)$ and $b \to c(\bar{u}s)$ transitions. As shown in Fig. 1, within the SM the $B^- \to DK^-$ are pure 'tree' decays. The effective Hamiltonian of this transition is given by

$$H_{\text{eff}}^{b \to u(\bar{c}s)} = \frac{G_{\text{F}}}{\sqrt{2}} V_{ub} V_{cs}^* \left[C_1(\mu) Q_1^u + C_2(\mu) Q_2^u \right] \,, \tag{12}$$

where C_i and Q_i^u are the Wilson coefficients, and the operators of this transition are renormalized at the scale μ with

$$\begin{aligned} Q_1^u &= \left(\bar{u}^\alpha \gamma_\mu L b^\alpha\right) \left(\bar{s}^\beta \gamma^\mu L c^\beta\right) \,, \\ Q_2^u &= \left(\bar{u}^\alpha \gamma_\mu L b^\beta\right) \left(\bar{s}^\beta \gamma^\mu L c^\alpha\right) \,, \end{aligned} \tag{13}$$

where $L = (1 - \gamma_5)$. The effective Hamiltonian for the $b \to c(\bar{u}s)$ transition can be obtained from the effective Hamiltonian in (12) by exchanging $u \leftrightarrow c$. The SM results for the corresponding Wilson coefficients are

$$\begin{aligned} C_1(m_W) &= 1 - \frac{11}{6} \frac{\alpha_{\text{s}}}{4\pi} \,, \\ C_2(m_W) &= \frac{14\alpha_{\text{s}}}{16\pi} \,. \end{aligned} \tag{14}$$

However, due to the QCD renormalization to the scale $\mu \simeq m_b$, C_1 and C_2 get mixed, as will be discussed in more detail in the next section, and one finds

$$\begin{aligned} C_1(\mu) &= 1.07 \,, \\ C_2(\mu) &= -0.17 \,. \end{aligned} \tag{15}$$

To evaluate the SM results to the decay amplitude of $B^- \to DK^-$, we have to determine the matrix elements for the operators $Q_{1,2}^{u,c}$. A detailed analysis for the matrix elements will be given in the next section. Here, we just give the matrix elements for these four operators in naive factorization:

$$\begin{aligned} \left\langle \bar{D}^0 K^- \left| Q_1^u \right| B^- \right\rangle &= -\frac{X}{3} \,, \\ \left\langle \bar{D}^0 K^- \left| Q_2^u \right| B^- \right\rangle &= -X \,, \end{aligned} \tag{16}$$

$$\begin{aligned} \left\langle D^0 K^- \left| Q_1^c \right| B^- \right\rangle &= -\frac{1}{3} X - Y \,, \\ \left\langle D^0 K^- \left| Q_2^c \right| B^- \right\rangle &= -X - \frac{1}{3} Y \,, \end{aligned} \tag{17}$$

where

$$\begin{aligned} X &= \mathrm{i} F_0^{B \to K}\left(m_D^2\right) f_D \left(m_B^2 - m_K^2\right) \,, \\ Y &= \mathrm{i} F_0^{B \to D}\left(m_K^2\right) f_K \left(m_B^2 - m_D^2\right) \,. \end{aligned} \tag{18}$$

There are two comments in order. i) The naive factorization can not determine the strong phases; therefore, in our analysis we consider these phases as free parameters. ii) As mentioned above, the factorized matrix element $\left\langle \bar{D}^0 K^- \left| (\bar{s}\gamma^\mu L c) \right| 0 \right\rangle \left\langle 0 \left| \bar{u}\gamma_\mu L b \right| B^- \right\rangle$ corresponding to an annihilation process is suppressed as shown in [6, 7] and can be neglected. Therefore,

$$A^{\text{SM}}(B^- \to \bar{D}^0 K^-) = -\frac{G_{\text{F}}}{\sqrt{2}} V_{ub} V_{cs}^* X \left(\frac{C_1}{3} + C_2 \right) \,, \tag{19}$$

and

$$\begin{aligned} &A^{\text{SM}}(B^- \to D^0 K^-) \\ &= -\frac{G_{\text{F}}}{\sqrt{2}} V_{cb} V_{us}^* \left[X \left(\frac{C_1}{3} + C_2 \right) + Y \left(C_1 + \frac{C_2}{3} \right) \right] \,. \end{aligned} \tag{20}$$

Fixing the hadronic parameters as follows: $f_D = 0.2$, $f_K = 0.16$, $F_0^{B \to D} = 0.34$, $F_0^{B \to K} = 0.62$, the meson masses become $m_K = 0.49$, $m_D = 1.86$, and $m_B = 5.278$ GeV. One finds

$$r_B \simeq 0.05 \,. \tag{21}$$

Note that it is customarily assumed that with a large uncertainty, the SM prediction for r_B may be much larger than the above value (it can be $\mathcal{O}(0.1)$ [9, 10]). Here we will use the value that we obtained in (21) as a typical value for the SM contribution. In order to have a general picture of the SM predictions for the CP asymmetries $A_{\pm}$ and the branching ratios $R_{\pm}$, we plot in Fig. 2 $R_{\pm}$ versus $A_{\pm}$. Here, we vary the parameter δ_B in the range $[0, \pi]$ and the angle γ is also considered to be between 0 and π.

As can be seen from the results in Fig. 2, the SM predictions for the branching ratios $R_{\pm}$ are within the 1σ experimental range. However, the results for the CP asymmetry A_+ are below its experimental lower bound. Also, the SM leads to a negative CP asymmetry A_- which is still consistent with its experimental results in (1), due to the large

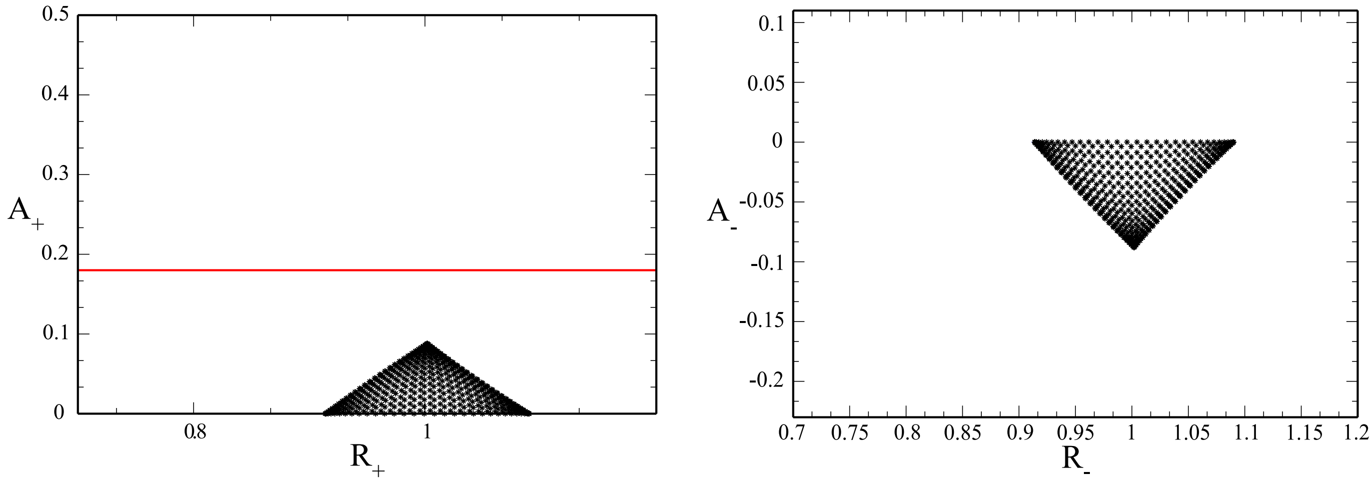

Fig. 2. R_+ versus A_+ and R_- versus A_- within the Standard Model. The horizontal line in the left figure represents the lower bound of A_+ at 1σ experimental range

uncertainties in this measurement. Therefore, more precise measurements would be very important for analyzing the SM predictions for $R_\pm$ and $A_\pm$ and, hence, in determining the value of the angle γ.

3 SUSY contributions to $b \to u(\bar{c}s)$ and $b \to c(\bar{u}s)$

The crucial point to note from Sect. 2 is that the SM contributions to the amplitudes of the $b \to u(\bar{c}s)$ and $b \to c(\bar{u}s)$ transitions are suppressed by the CKM factors $V_{ub} \simeq \mathcal{O}(10^{-3})$ and $V^*_{us}V_{cb} \simeq \mathcal{O}(10^{-3})$, respectively. Therefore, it may be possible to have a comparable effect from new physics at one loop level which can compete with the SM tree level contribution. In this section we study the supersymmetric contributions to the $b \to u(\bar{c}s)$ and $b \to c(\bar{c}s)$ transitions. In this case, the effective Hamiltonian $H^{\Delta C=1}_{\text{eff}}$ for $b \to u(\bar{c}s)$ can be expressed as

$$H^{\Delta C=1}_{\text{eff}} = \sum_{i=1}^{10} \left(C^u_i(\mu) Q^u_i(\mu) + \tilde{C}^u_i(\mu) \tilde{Q}^u_i(\mu) \right) , \quad (22)$$

where the C^u_i are the Wilson coefficients and the Q^u_i are the relevant local operators at low energy scale, $\mu \simeq m_b$. The operators Q^u_i are given by

$$\begin{aligned}
Q^u_1 &= \left(\bar{u}^\alpha \gamma_\mu L b^\alpha\right) \left(\bar{s}^\beta \gamma^\mu L c^\beta\right) , \\
Q^u_2 &= \left(\bar{u}^\alpha \gamma_\mu L b^\beta\right) \left(\bar{s}^\beta \gamma^\mu L c^\alpha\right) , \\
Q^u_3 &= \left(\bar{u}^\alpha \gamma_\mu L b^\alpha\right) \left(\bar{s}^\beta \gamma^\mu R c^\beta\right) \\
Q^u_4 &= \left(\bar{u}^\alpha \gamma_\mu L b^\beta\right) \left(\bar{s}^\beta \gamma^\mu R c^\alpha\right) , \\
Q^u_5 &= (\bar{u}^\alpha L b^\alpha)(\bar{s}^\beta L c^\beta) , \\
Q^u_6 &= (\bar{u}^\alpha L b^\beta)(\bar{s}^\beta L c^\alpha) , \\
Q^u_7 &= (\bar{u}^\alpha L b^\alpha)(\bar{s}^\beta R c^\beta) , \\
Q^u_8 &= (\bar{u}^\alpha L b^\beta)(\bar{s}^\beta R c^\alpha) , \\
Q^u_9 &= \left(\bar{u}^\alpha \sigma_{\mu\nu} L b^\alpha\right) \left(\bar{s}^\beta \sigma^{\mu\nu} L c^\beta\right) , \\
Q^u_{10} &= \left(\bar{u}^\alpha \sigma_{\mu\nu} L b^\beta\right) \left(\bar{s}^\beta \sigma^{\mu\nu} L c^\alpha\right) , \quad (23)
\end{aligned}$$

where α and β refer to the color indices. L and R are given by $(1 \mp \gamma_5)$, respectively, and $\sigma^{\mu\nu} = \frac{\mathrm{i}}{2}[\gamma^\mu, \gamma^\nu]$. The operators $\tilde{Q}^u_i$ are obtained from Q^u_i by the chirality exchange, $L \leftrightarrow R$. In the SM, the coefficients $\tilde{C}^u_i$ identically vanish, while in SUSY models they receive contributions from both gluino and chargino exchanges. The corresponding operators for $b \to c(\bar{u}s)$ can be obtained from the above expression by exchanging $u \leftrightarrow c$.

The dominant SUSY contribution to the $b \to u(\bar{c}s)$ transition can be generated through the box diagrams with gluino exchange, as in Fig. 3, and chargino exchange, as in Fig. 4. From these figures, one can see that the $b \to u(\bar{c}s)$ transition is based on two topologically distinct box diagrams only for gluino or chargino exchange. This is unlike the $b \to d$ and $b \to s$ transitions that contribute to B–$\bar{B}$ mixing, where four topologically distinct box diagrams are included [11]. Therefore, it is expected that the Wilson coefficients for this process are different from those obtained in the literature for the $b \to s$ transition. It is also worth mentioning that contributions through penguin diagrams to these transitions are always hybrid (i.e., contain internal SUSY and SM particles). Therefore, they are suppressed by V_{ub} in addition to the usual loop suppression factor; hence they are much smaller than the pure SM or pure SUSY contributions. Thus, the Wilson coefficients at m_W scale can be expressed as follows:

$$C^u_i = (C^u_i)^{\text{SM}} + (C^u_i)^{\tilde{g}} + (C^u_i)^{\tilde{\chi}} , \quad (24)$$

We evaluate the SUSY contributions to the Wilson coefficients by using the mass insertion approximation. The mass insertion approximation is a quite useful method for performing a model independent analysis of flavor changing processes in general SUSY models. In our analysis we set to zero the contributions that are proportional to the Yukawa coupling of light quarks. Also, we use the approximation of retaining only terms proportional to order λ. In the case of gluino exchange all the above operators give significant contributions, and the corresponding Wilson coef-

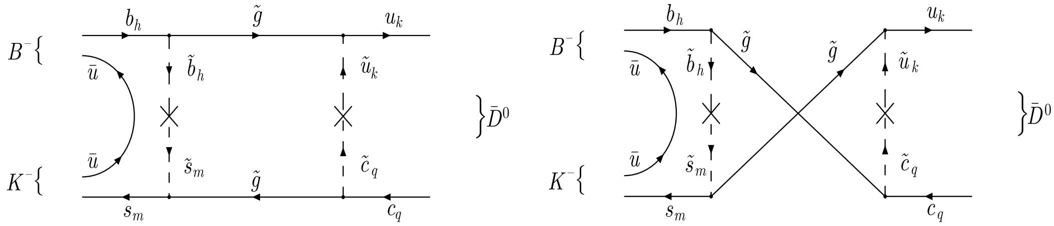

Fig. 3. Box diagrams for $B^- \to K^- \bar{D}^0$ $(b \to u(\bar{c}s)$ transition) with gluino exchanges, where $h, k, m, n = \{\mathrm{L}, \mathrm{R}\}$

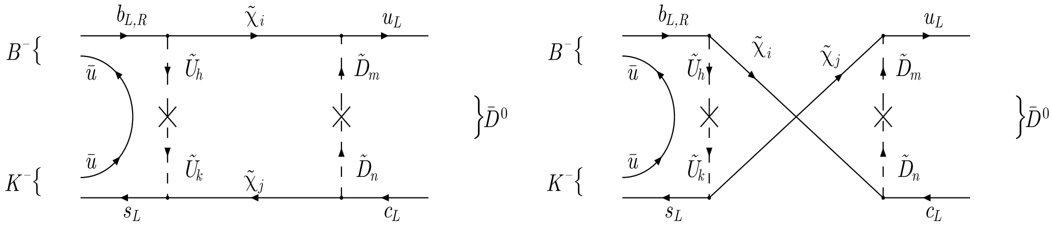

Fig. 4. Box diagrams for $B^- \to K^- \bar{D}^0$ $(b \to u(\bar{c}s)$ transition) with chargino exchanges, where $U = \{u, c, t\}$, $D = \{d, s, b\}$ and $h, k, m, n = \{\mathrm{L}, \mathrm{R}\}$

ficients are given by

$$C_1^{\tilde{g}}(m_W) = \frac{\alpha_s^2}{48\tilde{m}^2} \left(\delta_{LL}^d\right)_{23} (\delta_{LL}^u)_{12} \left[7\tilde{f}_6(x) - 4x f_6(x)\right], \quad (25)$$

$$C_2^{\tilde{g}}(m_W) = \frac{\alpha_s^2}{144\tilde{m}^2} \left(\delta_{LL}^d\right)_{23} (\delta_{LL}^u)_{12} \left[\tilde{f}_6(x) + 20x f_6(x)\right], \quad (26)$$

$$C_3^{\tilde{g}}(m_W) = \frac{5\alpha_s^2}{48\tilde{m}^2} \left(\delta_{RL}^d\right)_{23} (\delta_{LR}^u)_{12} \tilde{f}_6(x), \quad (27)$$

$$C_4^{\tilde{g}}(m_W) = \frac{11\alpha_s^2}{144\tilde{m}^2} \left(\delta_{RL}^d\right)_{23} (\delta_{LR}^u)_{12} \tilde{f}_6(x), \quad (28)$$

$$C_5^{\tilde{g}}(m_W) = \frac{2\alpha_s^2}{3\tilde{m}^2} \left(\delta_{RL}^d\right)_{23} (\delta_{RL}^u)_{12} x f_6(x), \quad (29)$$

$$C_6^{\tilde{g}}(m_W) = \frac{-\alpha_s^2}{9\tilde{m}^2} \left(\delta_{RL}^d\right)_{23} (\delta_{RL}^u)_{12} x f_6(x), \quad (30)$$

$$C_7^{\tilde{g}}(m_W) = \frac{\alpha_s^2}{12\tilde{m}^2} \left(\delta_{RR}^d\right)_{23} (\delta_{LL}^u)_{12} \left[-\tilde{f}_6(x) + 7x f_6(x)\right], \quad (31)$$

$$C_8^{\tilde{g}}(m_W) = \frac{\alpha_s^2}{36\tilde{m}^2} \left(\delta_{RR}^d\right)_{23} (\delta_{LL}^u)_{12} \left[5\tilde{f}_6(x) + x f_6(x)\right], \quad (32)$$

$$C_9^{\tilde{g}}(m_W) = -\frac{\alpha_s^2}{48\tilde{m}^2} \left(\delta_{RL}^d\right)_{23} (\delta_{RL}^u)_{12} x f_6(x), \quad (33)$$

$$C_{10}^{\tilde{g}}(m_W) = \frac{5\alpha_s^2}{144\tilde{m}^2} \left(\delta_{RL}^d\right)_{23} (\delta_{RL}^u)_{12} x f_6(x), \quad (34)$$

where $x = m_{\tilde{g}}^2/\tilde{m}^2$. $m_{\tilde{g}}$ is the gluino mass, and $\tilde{m}^2$ is an average squark mass. The functions $f_6(x)$ and $\tilde{f}_6(x)$ are the same as the loop function obtained in the case of $b \to d(\bar{q}q)$ and are given by

$$f_6(x) = \frac{6(1+3x)\ln x + x^3 - 9x^2 - 9x + 17}{6(x-1)^5}, \quad (35)$$

$$\tilde{f}_6(x) = \frac{6x(1+x)\ln x - x^3 - 9x^2 + 9x + 1}{3(x-1)^5}. \quad (36)$$

The Wilson coefficients $\tilde{C}_i^{\tilde{g}}$ are simply obtained by interchanging $L \leftrightarrow R$ in the mass insertions appearing in $C_i^{\tilde{g}}$. The above Wilson coefficients are due to the gluino exchange of the $b \to u$ transition; the corresponding coefficients for the $b \to c$ transition can be obtained by changing the mass insertions $(\delta_{AB}^u)_{12}$ to $(\delta_{AB}^u)_{21}$, where $\{A, B\} = \{L, R\}$.

Note that the discrepancy between the above SUSY Wilson coefficients of the $b \to u$ transition and those of the $b \to d$ or $b \to s$ $\Delta B = 2$ transition is due to the following reasons.

1. The $b \to u$ transition is based, as mentioned above, on two distinct box diagrams only in contrast of the $\Delta B = 2$ transition where four distinct box diagrams are involved.
2. All the external quarks in the box diagrams of $b \to u(\bar{c}s)$ are different; therefore, one cannot use a Fierz transformation to relate any operator with the other unlike the case in $\Delta B = 2$. For instance, in B_d–$\bar{B}_d$ mixing the operator $Q_2 = (\bar{d}^\alpha \gamma_\mu L b^\beta)(\bar{d}^\beta \gamma_\mu L b^\alpha)$ is equivalent to the operator $(\bar{d}^\alpha \gamma_\mu L b^\alpha)(\bar{d}^\beta \gamma_\mu L b^\beta)$. In this case, the Wilson coefficients C_1 and C_2 in (25) and (26) are combined together and leads to the usual $\Delta B = 2$ Wilson coefficient [11]: $C_1 \propto \alpha_s/108\tilde{m}^2(24x f_6(x) + 66\tilde{f}_6(x))$. In this respect, it is clear that the expression used in (9) in [5] for $H_{\text{eff}}^{b \to u(\bar{c}s)}$ is incorrect.

Now let us turn to the chargino contributions to the effective Hamiltonian in (22) in the mass insertion approximation. The leading diagrams are illustrated in Fig. 4, where the cross in the middle of the squark propagator represents a single mass insertion. In the above mentioned approximation, where we neglect contributions proportional to the light quark masses, one finds that the relevant chargino exchange affects only the operator Q_1, as in the SM, and the corresponding Wilson coefficient is given by

$$\begin{aligned} C_1^{\tilde{\chi}}(m_W) &= \frac{\alpha\sqrt{\alpha}}{16\tilde{m}^2} \Big[\sqrt{\alpha} V_{i1}^* V_{j1} U_{i1} U_{j1}^* \left(\delta_{LL}^d\right)_{12} \left((\delta_{LL}^u)_{23} + \lambda\, (\delta_{LL}^u)_{13}\right) \\ &\quad + \frac{y_t}{\sqrt{4\pi}} U_{i1} U_{j1}^* V_{j1} V_{i2}^* \left(\delta_{LL}^d\right)_{12} \left((\delta_{LR}^u)_{23} + \lambda\, (\delta_{LR}^u)_{13}\right)\Big] \\ &\quad \times \left(L_2(x_i, x_j) - 2L_0(x_i, x_j)\right), \end{aligned} \quad (37)$$

where $\alpha = g^2/4\pi$ and g is the $SU(2)$ gauge coupling constant. The λ parameter stands for the Cabibbo mixing, i.e., $\lambda = 0.22$. The U_{ij} and V_{ij} are the unitary matrices that diagonalize the chargino mass matrix, and y_t is the top Yukawa coupling. We have $x_i = m^2_{\tilde{\chi}_i}/\tilde{m}^2$, and the functions $L_0(x,y)$ and $L_2(x,y)$ are given by [11]

$$\begin{aligned} L_0(x,y) &= \sqrt{xy}\frac{xh_0(x)-yh_0(y)}{x-y}\,, \\ h_0(x) &= \frac{-11+7x-2x^2}{(1-x)^3} - \frac{6\ln x}{(1-x)^4}\,, \\ L_2(x,y) &= \frac{xh_2(x)-yh_2(y)}{x-y}\,, \\ h_2(x) &= \frac{2+3x-x^2}{(1-x)^3} + \frac{6x\ln x}{(1-x)^4}\,. \end{aligned} \tag{38}$$

Finally, we have also neglected the small contributions from the box diagrams where both gluino and chargino is exchanged as in [12, 13].

To obtain the Wilson coefficients at the scale m_b one has to solve the corresponding renormalization group equations. The solution is generally expressed as

$$C_i(m_b) = \sum_j U_{ij}(m_b, m_W) C_j(m_W), \tag{39}$$

where $U_{ij}(m_b, m_W)$ is the evolution matrix given by the 8×8 anomalous dimension matrix of leading order (LO) corrections in QCD [14]. Note that we have not included the operators $Q_{9,10}$, since they have zero matrix elements at LO, and also they do not mix with the other operators in the evolution from the m_W scale down to the m_b scale. We have

$$U(m_b, m_W) = \hat{V}\left(\left[\frac{\alpha_s(m_W)}{\alpha_s(m_b)}\right]^{-\frac{\gamma^{(0)}}{2\beta_0}}\right)_D \hat{V}^{-1}, \tag{40}$$

where $\hat{V}$ diagonalizes the $\hat{\gamma}^{(0)\mathrm{T}}$:

$$\hat{\gamma}^{(0)}_D = \hat{V}^{-1}\hat{\gamma}^{(0)\mathrm{T}}\hat{V}\,, \tag{41}$$

and $\gamma^{(0)}$ is for the diagonal elements of $\hat{\gamma}^{(0)}_D$. The value of β_0 is given by $\beta_0 = \frac{11}{3}N_c - \frac{2}{3}f$ where N_c is the number of colors, and f is number of active flavors. Finally, the anomalous dimension matrix $\hat{\gamma}^0$ at the leading order is given by

$$\hat{\gamma}^0 = \begin{pmatrix} -2 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -16 \end{pmatrix}. \tag{42}$$

As can be seen from the above matrix, the mixing between different operators is divided into blocks. Each block contains two operators (Q_i, Q_{i+1}), $i = 1,3,4,7$, there being no mixing between different blocks [15].

Let us now consider the evaluation of the hadronic matrix elements of the above operators which represents the most uncertain part in this calculation. In the limit of neglecting QCD corrections and $m_b \gg \Lambda_{\mathrm{QCD}}$, the hadronic matrix elements of $B^- \to DK^-$ decay can be factorized. The hadronic matrix elements for the operators Q^u_i are given by

$$\langle D^0K^-|Q^u_i|B^-\rangle = 0 \tag{43}$$

and

$$\begin{aligned} \langle \bar{D}^0K^-|Q^u_1|B^-\rangle &= -\frac{X}{3}\,, \\ \langle \bar{D}^0K^-|Q^u_2|B^-\rangle &= -X\,, \\ \langle \bar{D}^0K^-|Q^u_3|B^-\rangle &= \frac{2m_D^2}{3(m_b-m_s)(m_u+m_c)}X\,, \\ \langle \bar{D}^0K^-|Q^u_4|B^-\rangle &= \frac{2m_D^2}{(m_b-m_s)(m_u+m_c)}X\,, \\ \langle \bar{D}^0K^-|Q^u_5|B^-\rangle &= \langle \bar{D}^0K^-|Q^u_6|B^-\rangle = 0\,, \\ \langle \bar{D}^0K^-|Q^u_7|B^-\rangle &= \frac{X}{6}\,, \\ \langle \bar{D}^0K^-|Q^u_8|B^-\rangle &= \frac{X}{2}\,, \\ \langle \bar{D}^0K^-|Q^u_9|B^-\rangle &= \langle \bar{D}^0K^-|Q^u_{10}|B^-\rangle = 0\,, \end{aligned} \tag{44}$$

while the hadronic matrix elements for the operators Q^c_i are given as follows:

$$\langle \bar{D}^0K^-|Q^c_i|B^-\rangle = 0\,, \tag{45}$$

and

$$\begin{aligned} \langle D^0K^-|Q^c_1|B^-\rangle &= -Y - \frac{1}{3}X\,, \\ \langle D^0K^-|Q^c_2|B^-\rangle &= -\frac{1}{3}Y - X\,, \\ \langle D^0K^-|Q^c_3|B^-\rangle &= Y + \frac{2m_D^2}{3(m_b-m_s)(m_u+m_c)}X\,, \\ \langle D^0K^-|Q^c_4|B^-\rangle &= \frac{1}{3}Y + \frac{2m_D^2}{(m_b-m_s)(m_u+m_c)}X\,, \\ \langle D^0K^-|Q^c_5|B^-\rangle &= \langle D^0K^-|Q^c_6|B^-\rangle = 0\,, \\ \langle D^0K^-|Q^u_7|B^-\rangle &= -\frac{m_K^2}{(m_b-m_c)(m_u+m_s)}Y + \frac{1}{6}X\,, \\ \langle D^0K^-|Q^c_8|B^-\rangle &= -\frac{1}{3}\frac{m_K^2}{(m_b-m_c)(m_u+m_s)}Y + \frac{1}{2}X\,, \\ \langle D^0K^-|Q^c_9|B^-\rangle &= \langle D^0K^-|Q^c_{10}|B^-\rangle = 0 \end{aligned} \tag{46}$$

where X and Y are given in (18).

Having evaluated the SUSY contributions to the Wilson coefficients and having determined the hadronic matrix elements of the relevant operators, one can analyze the decay amplitude of $B^- \to DK^-$ and study the SUSY effect on the CP asymmetries $A_\pm$ and the branching ratios $R_\pm$. As can be observed, the Wilson coefficients depend on several mass insertions, which are in general complex and provide new sources for the CP violation beyond the

SM phase in the CKM mixing matrix. These new CP violating phases may contribute significantly to the $b \to u$ transition and affect the determination of the angle γ. Nevertheless, one should be very careful with the constraints imposed on these parameters. In fact, the dominant gluino contributions depend on the mass insertions: $(\delta^u_{AB})_{12}$ and $\left(\delta^d_{AB}\right)_{23}$. The mass insertions $\left(\delta^d_{AB}\right)_{23}$ are constrained by the experimental results for the branching ratio of $B \to X_s\gamma$ [16–19]. These constraints are very weak on the LL or RR mass insertion and more stronger for LR or RL mass insertions. Concerning the mass insertion $(\delta^u_{AB})_{12}$, the chargino contributions to the K^0–$\bar{K}^0$ system impose constraint on the LL mass insertion only [20]. However, D^0–$\bar{D}^0$ mixing may induce more strange constraints on both the $LL(RR)$ and the $LR(RL)$ mass insertions. Therefore, before we proceed to analyzing the decay amplitude of $B^- \to DK^-$, we will take a short detour and give a detailed analysis for the SUSY contributions to D^0–$\bar{D}^0$ mixing and revise the corresponding constraints on the $(\delta^u_{AB})_{12}$ mass insertions.

4 Constraints from D^0–$\bar{D}^0$ mixing

We start this section by summarizing the SM results for the D^0–$\bar{D}^0$ mixing; then we consider the supersymmetric contribution to the effective Hamiltonian for the $\Delta C = 2$ transitions given by gluino and chargino box exchanges.

In the D^0 and $\bar{D}^0$ systems, the flavor eigenstates are given by $D^0 = (\bar{u}c)$ and $\bar{D}^0 = (u\bar{c})$. The formalism for D^0–$\bar{D}^0$ mixing is the same as the one used for K^0–$\bar{K}^0$ and B^0–$\bar{B}^0$ mixing. The mass eigenstates are given in terms of the weak eigenstates by

$$D_{1,2} = pD^0 \pm q\bar{D}^0\,, \tag{47}$$

where the ratio q/p can be written in terms of the off-diagonal element of the mass matrix: $q/p = \sqrt{M^*_{12}/M_{12}}$, and $q/p \neq 1$ is an indication for the CP violation through mixing. The strength of the D^0–$\bar{D}^0$ mixing is described by the mass difference

$$\Delta M_D = M_{D_1} - M_{D_2}\,.$$

The present experimental results for ΔM_D is given by [21]

$$(\Delta M_D)_{\text{exp}} < 1.7 \times 10^{-13}\,\text{GeV}\,. \tag{48}$$

The CP asymmetry of the D^0 and $\bar{D}^0$ meson decay to a CP eigenstate f is given by

$$\begin{aligned} a_f(t) &= \frac{\Gamma(D^0 \to f) - \Gamma(\bar{D}^0 \to f)}{\Gamma(D^0 \to f) + \Gamma(\bar{D}^0 \to f)} \\ &= S_f \sin(\Delta M_D t) + C_f \cos(\Delta M_D t)\,, \end{aligned} \tag{49}$$

where S_f and C_f represent the mixing and direct CP asymmetry, respectively, and they are given by

$$\begin{aligned} S_f &= \frac{2\text{Im}\left[\frac{q}{p}\bar{\rho}(f)\right]}{|\bar{\rho}(f)|^2 + 1}\,, \\ C_f &= \frac{|\bar{\rho}(f)|^2 - 1}{|\bar{\rho}(f)|^2 + 1}\,. \end{aligned} \tag{50}$$

The parameter $\bar{\rho}(f)$ is defined by $\bar{\rho}(f) = \frac{\mathcal{A}(\bar{D}^0 \to f)}{\mathcal{A}(D^0 \to f)}$. Generically, the ΔM_D and S_f can be calculated by

$$\begin{aligned} \Delta M_D &= 2\left|\left\langle D^0 \left| H_{\text{eff}}^{\Delta C=2} \right| \bar{D}^0 \right\rangle\right|\,, \\ S_f &= \sin\left(\arg\left[\left\langle D^0 \left| H_{\text{eff}}^{\Delta C=2} \right| \bar{D}^0 \right\rangle\right]\right)\,. \end{aligned} \tag{51}$$

Here $H_{\text{eff}}^{\Delta C=2}$ is the effective Hamiltonian responsible for the $\Delta C = 2$ transition. In the framework of the SM, this transition occurs via a box diagram in which two virtual down quarks and two virtual W bosons are exchanged. $\Delta M_D^{\text{SM}} = 2|\langle D^0|(H_{\text{eff}}^{\Delta C=2})^{\text{SM}}|\bar{D}^0\rangle|$ is given by [22]

$$\begin{aligned} \Delta M_D^{\text{SM}} &\simeq \frac{G_{\text{F}}^2}{3\pi^2} M_D^2 f_D^2 \left|V^*_{cs} V_{cd}\right|^2 \frac{\left(m_s^2 - m_d^2\right)^2}{m_c^2} \\ &\simeq 1.4 \times 10^{-18}\,\text{GeV}\,. \end{aligned} \tag{52}$$

As can be seen from this expression, the SM predicts a very small D^0–$\bar{D}^0$ mixing. Note that, in the above estimation for ΔM_D^{SM}, the b-quark contribution has been neglected, since it is much smaller due to the CKM suppression. Also, the CP violation is absent in the mixing and in the dominant tree level decay due to the first two generations being involved only.

In supersymmetric theories, the dominant contributions to the off-diagonal entry in the D^0 meson mass matrix, $\mathcal{M}_{12} = \left\langle D^0 \left| H_{\text{eff}}^{\Delta C=2} \right| \bar{D}^0 \right\rangle$, is given by

$$\mathcal{M}_{12} = \mathcal{M}_{12}^{\text{SM}} + \mathcal{M}_{12}^{\tilde{g}} + \mathcal{M}_{12}^{\tilde{\chi}^+}\,, \tag{53}$$

where $\mathcal{M}_{12}^{\tilde{g}}$, and $\mathcal{M}_{12}^{\tilde{\chi}^+}$ correspond to the gluino and chargino contributions, respectively. The effect of supersymmetry can be parameterized as follows:

$$r_c^2 \text{e}^{2\text{i}\theta_c} = \frac{\mathcal{M}_{12}}{\mathcal{M}_{12}^{\text{SM}}}\,, \tag{54}$$

where $\Delta M_D = 2\left|\mathcal{M}_{12}^{\text{SM}}\right| r_c^2$ and $2\theta_c = \arg\left(1 + \frac{\mathcal{M}_{12}^{\text{SUSY}}}{\mathcal{M}_{12}^{\text{SM}}}\right)$. As in the case of the K^0 and B^0 systems, the most general effective Hamiltonian for the $\Delta C = 2$ processes, induced by gluino and chargino exchanges through box diagrams, can be expressed as

$$H_{\text{eff}}^{\Delta C=2} = \sum_{i=1}^{5} C_i(\mu) Q_i(\mu) + \sum_{i=1}^{3} \tilde{C}_i(\mu) \tilde{Q}_i(\mu) + \text{h.c.}\,, \tag{55}$$

where $C_i(\mu)$, $\tilde{C}_i(\mu)$ and $Q_i(\mu)$, $\tilde{Q}_i(\mu)$ are the Wilson coefficients and operators respectively renormalized at the scale

μ, with

$$\begin{aligned} Q_1 &= \bar{u}^{\alpha}_{\mathrm{L}}\gamma_\mu c^{\alpha}_{\mathrm{L}}\bar{u}^{\beta}_{\mathrm{L}}\gamma_\mu c^{\beta}_{\mathrm{L}}\,,\\ Q_2 &= \bar{u}^{\alpha}_{\mathrm{R}} c^{\alpha}_{\mathrm{L}}\bar{u}^{\beta}_{\mathrm{R}} c^{\beta}_{\mathrm{L}}\,,\\ Q_3 &= \bar{u}^{\alpha}_{\mathrm{R}} c^{\beta}_{\mathrm{L}}\bar{u}^{\beta}_{\mathrm{R}} c^{\alpha}_{\mathrm{L}}\,,\\ Q_4 &= \bar{u}^{\alpha}_{\mathrm{R}} c^{\alpha}_{\mathrm{L}}\bar{u}^{\beta}_{\mathrm{L}} c^{\beta}_{\mathrm{R}}\,,\\ Q_5 &= \bar{u}^{\alpha}_{\mathrm{R}} c^{\beta}_{\mathrm{L}}\bar{u}^{\beta}_{\mathrm{L}} c^{\alpha}_{\mathrm{R}}\,. \end{aligned} \tag{56}$$

In addition, the operators $\tilde{Q}_{1,2,3}$ are obtained from $Q_{1,2,3}$ by exchanging $L \leftrightarrow R$.

In the case of gluino exchange all the above operators give significant contributions, and the corresponding Wilson coefficients are given by [23]

$$\begin{aligned} C_1^{\tilde{g}}(m_W) &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\left(24 x f_6(x) + 66\tilde{f}_6(x)\right)(\delta^u_{12})^2_{LL}\,,\\ C_2^{\tilde{g}}(m_W) &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2} 204 x f_6(x)\,(\delta^u_{12})^2_{RL}\,,\\ C_3^{\tilde{g}}(m_W) &= \frac{\alpha_s^2}{216 m_{\tilde{q}}^2} 36 x f_6(x)\,(\delta^u_{12})^2_{RL}\,,\\ C_4^{\tilde{g}}(m_W) &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\left[\left(504 x f_6(x) - 72\tilde{f}_6(x)\right)\right.\\ &\quad \left.\times (\delta^u_{12})_{LL}(\delta^u_{12})_{RR} - 132\tilde{f}_6(x)(\delta^u_{12})_{LR}(\delta^u_{12})_{RL}\right],\\ C_5^{\tilde{g}}(m_W) &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\left[\left(24 x f_6(x) + 120\tilde{f}_6(x)\right)\right.\\ &\quad \left.\times (\delta^u_{12})_{LL}(\delta^u_{12})_{RR} - 180\tilde{f}_6(x)(\delta^u_{12})_{LR}(\delta^u_{12})_{RL}\right], \end{aligned} \tag{57}$$

where $x = m_{\tilde{g}}^2/\tilde{m}^2$ and $\tilde{m}^2$ is an average squark mass. The functions $f_6(x)$ and $\tilde{f}_6(x)$ are given in (35) and (36)).The Wilson coefficients $\tilde{C}_{1-3}$ are simply obtained by interchanging $L \leftrightarrow R$ in the mass insertions appearing in C_{1-3}.

In the case of the chargino exchange the operator Q_1 only gives a significant contribution [20]. At the first order in the mass insertion approximation, the Wilson coefficient $C_1^{\chi}(m_W)$ is given by

$$C_1^{\chi}(m_W) = \frac{g^4}{768\pi^2\tilde{m}^2}\sum_{i,j}|V_{i1}|^2\,|V_{j1}|^2\left(\delta^d_{21}\right)^2_{LL} L_2(x_i,x_j)\,, \tag{58}$$

where $x_i = m^2_{\tilde{\chi}^+_i}/\tilde{m}^2$, and the function $L_2(x,y)$ is as given in (38).

As usual, the Wilson coefficients $C_i(\mu)$ are related to $C_i(m_W)$ by [24]

$$C_r(\mu) = \sum_i\sum_s\left(b_i^{(r,s)} + \eta c_i^{(r,s)}\right)\eta^{a_i} C_s(m_W)\,, \tag{59}$$

where $\eta = \alpha_S(m_W)/\alpha_S(\mu)$, and the coefficients $b_i^{(r,s)}$, $c_i^{(r,s)}$ and a_i appearing in (59) can be found in [24]. Also the matrix elements of the operators Q_i in the vacuum insertion approximation are given by [23]

$$\begin{aligned} \langle D^0|Q_1|\bar{D}^0\rangle &= \frac{1}{3} M_D f_D^2\,,\\ \langle D^0|Q_2|\bar{D}^0\rangle &= -\frac{5}{24}\left(\frac{M_D}{m_c+m_u}\right)^2 M_D f_D^2\,,\\ \langle D^0|Q_3|\bar{D}^0\rangle &= \frac{1}{24}\left(\frac{m M_D}{m_c+m_u}\right)^2 M_D f_D^2\,,\\ \langle D^0|Q_4|\bar{D}^0\rangle &= \left[\frac{1}{24} + \frac{1}{4}\left(\frac{M_D}{m_c+m_u}\right)^2\right] M_D f_D^2\,,\\ \langle D^0|Q_5|\bar{D}^0\rangle &= \left[\frac{1}{8} + \frac{1}{12}\left(\frac{M_D}{m_c+m_u}\right)^2\right] M_D f_D^2\,. \end{aligned} \tag{60}$$

The same results are also valid for the corresponding operators $\tilde{Q}_i$, since strong interactions preserve parity.

We now discuss the results of the SUSY contribution to D^0–$\bar{D}^0$ mixing. It is worth mentioning that the mass insertions $\left(\delta^d_{AB}\right)_{12}$ are strongly constrained by the experimental limits of K^0–$\bar{K}^0$ mixing. In particular, the ΔM_K upper bound implies that $\left|\left(\delta^d_{LL}\right)^2_{12}\right| \le 10^{-4}$ [23]. Therefore, the chargino contribution to ΔM_D becomes very suppressed and can be neglected with respect to the gluino contributions which depend on the less constrained mass insertions $(\delta^u_{AB})_{12}$. As an example, we present the gluino contribution to ΔM_D, with $m_{\tilde{g}} \simeq m_{\tilde{q}} \simeq 500$ GeV:

$$\begin{aligned} \frac{\Delta M_D^{\mathrm{SUSY}}}{1.7\times 13} \simeq \Big| & 33.4\,(\delta^u_{LL})^2_{12} + 1733.6\,(\delta^u_{LR})^2_{12}\\ & - 3178.5\,(\delta^u_{LR})_{12}(\delta^u_{RL})_{12} + 1733.6\,(\delta^u_{RL})^2_{12}\\ & - 12946.9\,(\delta^u_{LL})_{12}(\delta^u_{RR})_{12} + 33.4\,(\delta^u_{RR})^2_{12}\Big|\\ & < 1. \end{aligned} \tag{61}$$

From this expression, we can see that the strongest constraint will be imposed on the product $(\delta^u_{LL})_{12}\,(\delta^u_{RR})_{12}$, while the constraint obtained on the individual mass insertion $(\delta^u_{LL})_{12}$ or $(\delta^u_{RR})_{12}$ is less stringent.

As usual in this kind of analysis, the most conservative constraints on the mass insertions can be obtained by considering the contribution due to a single mass insertion per time and set all other ones to zero. In Table 1, we present the results for the upper bounds on the relevant mass insertions from the experimental constraint on ΔM_D for $x = 1/4, 1$, and 4. We find that these bounds on $(\delta^u_{AB})_{12}$ are more stringent than those obtained from the chargino contribution to the K^0–$\bar{K}^0$ system in [20]. In fact, the $(\delta^u_{LR})_{12}$ and $(\delta^u_{RL})_{12}$ are completely unconstrained by the chargino contribution to K^0–$\bar{K}^0$ mixing. Therefore, their bounds in the above table are the only known constraints. However, we should mention that these constraints may be relaxed if one considers simultaneous contributions from more than one mass insertion. In this case, a possible cancellation may occur which reduces the SUSY contribution significantly and leaves room for a larger mass insertion.

Table 1. Upper bounds on $\sqrt{\left|(\delta^u_{AB})^2_{12}\right|}$ from $\Delta M_D < 1.7 \times 10^{-13}$ GeV, for $x = m^2_{\tilde{g}}/m^2_{\tilde{q}} = 1/4, 1, 4$

x	$\sqrt{\left|(\delta^u_{LL(RR)})^2_{12}\right|}$	$\sqrt{\left|(\delta^u_{LR(RL)})^2_{12}\right|}$	$\sqrt{\left|(\delta^u_{LL})_{12}(\delta^u_{RR})_{12}\right|}$	$\sqrt{\left|(\delta^u_{LR})_{12}(\delta^u_{RL})_{12}\right|}$
1/4	7.5×10^{-2}	2.1×10^{-2}	7.5×10^{-3}	1×10^{-2}
1	1.7×10^{-2}	2.4×10^{-2}	8.7×10^{-3}	1.7×10^{-2}
4	0.4	3.3×10^{-2}	1.2×10^{-2}	4×10^{-2}

Finally, we comment on the CP violation in this process. As emphasized above, the SM contribution to D^0–$\bar{D}^0$ mixing is real, since it is proportional to $V^*_{cs}V_{cd}$. Furthermore, it is much smaller than the dominant gluino contribution. Therefore the CP violating phase θ_c in (54) can be written as

$$\theta_c = \frac{1}{2}\arg\left(\frac{\mathcal{M}_{12}}{\mathcal{M}^{\rm SM}_{12}}\right) \simeq \frac{1}{2}\arg\left(\mathcal{M}^{\tilde{g}}_{12}\right) . \qquad (62)$$

In case $(\delta^u_{LR})_{12}$ gives a dominant contribution to $\mathcal{M}^{\tilde{g}}_{12}$, θ_c will be given by

$$\theta_c = \frac{1}{2}\arg\left((\delta^u_{LR})_{12}\right) \simeq \mathcal{O}(1) , \qquad (63)$$

which means that the CP asymmetry of D^0–$\bar{D}^0$ mixing, S_f, could be quite large. Therefore, one can conclude that the new physics in general and supersymmetry in particular could enhance the D^0–$\bar{D}^0$ mixing significantly.

5 Supersymmetric contribution to $R_\pm$ and $A_\pm$

In this section we study the supersymmetric contributions to the CP asymmetries and the branching ratios of $B^- \to DK^-$ decay in the following cases. 1) Negligible D^0–$\bar{D}^0$ mixing. 2) Large D^0–$\bar{D}^0$ mixing due to a possible significant SUSY contribution, as advocated in Sect. 4.

In general, applying the naive factorization approximation implies that the amplitudes $A(B^- \to DK^-)$ are given by

$$A(B^- \to D^0K^-) = \sum_{i=1}^{8}\left(C^c_i - \tilde{C}^c_i\right)\left\langle D^0K^-\left|Q^c_i\right|B^-\right\rangle \qquad (64)$$

and

$$A(B^- \to \bar{D}^0K^-) = \sum_{i=1}^{8}\left(C^u_i - \tilde{C}^u_i\right)\left\langle \bar{D}^0K^-\left|Q^u_i\right|B^-\right\rangle . \qquad (65)$$

The sign difference between the Wilson coefficients C_i and $\tilde{C}_i$ in the above equations is due to the fact that the initial and final states of the $B^- \to DK^-$ decays have opposite parity, and therefore $\langle DK^-|Q_i|B^-\rangle = -\langle DK^-|\tilde{Q}_i|B^-\rangle$ [25, 26].

5.1 $R_\pm$ and $A_\pm$ with negligible D^0–$\bar{D}^0$ mixing

In case of neglecting the effect of D^0–$\bar{D}^0$ mixing, it is useful to parameterize the SUSY contribution by introducing the ratio of the SM and SUSY amplitudes as follows:

$$\frac{A^{\rm SUSY}(B^- \to \bar{D}^0K^-)}{A^{\rm SM}(B^- \to \bar{D}^0K^-)} = R_1 e^{i(\phi_1-\gamma)}e^{i\delta_1} , \qquad (66)$$

and

$$\frac{A^{\rm SUSY}(B^- \to D^0K^-)}{A^{\rm SM}(B^- \to D^0K^-)} = R_2 e^{i\phi_2}e^{i\delta_2} , \qquad (67)$$

where R_i stands for the corresponding absolute value of $|A^{\rm SUSY}/A^{\rm SM}|$, the angles ϕ_i are the corresponding SUSY CP violating phases, and the $\delta_i = \delta^{\rm SM}_i - \delta^{\rm SUSY}_i$ are the strong phases. In this respect, our previous definition for the SM ratio of the amplitudes of $B^- \to \bar{D}^0K^-$ and $B^- \to D^0K^-$ in (7) will be generalized as follows:

$$\begin{aligned}&\frac{A(B^- \to \bar{D}^0K^-)}{A(B^- \to D^0K^-)}\\ &= \frac{A^{\rm SM}(B^- \to \bar{D}^0K^-) + A^{\rm SUSY}(B^- \to \bar{D}^0K^-)}{A^{\rm SM}(B^- \to D^0K^-) + A^{\rm SUSY}(B^- \to D^0K^-)}\\ &= r_B e^{i\delta_B}\left[\frac{e^{i\gamma} + R_1 e^{i\phi_1}}{1 + R_2 e^{i\phi_2}}\right] \equiv R_B e^{i\delta_B}e^{i\phi_B} ,\end{aligned} \qquad (68)$$

where

$$R_B = r_B\left|\frac{e^{i\gamma} + R_1 e^{i\phi_1}}{1 + R_2 e^{i\phi_2}}\right| ,$$

and

$$\phi_B = \arg\left[\frac{e^{i\gamma} + R_1 e^{i\phi_1}}{1 + R_2 e^{i\phi_2}}\right] . \qquad (69)$$

Note that, for simplicity, we have assumed that the SM and SUSY strong phases are equal. In this case, the ratios $R_\pm$ and the CP asymmetries $A_\pm$ take the form

$$R_\pm = 1 + R^2_B \pm 2R_B \cos\delta_B \cos\phi_B \qquad (70)$$

and

$$A_\pm = \frac{\pm 2R_B \sin\delta_B \sin\phi_B}{1 + R^2_B \pm 2R_B \cos\delta_B \cos\phi_B} . \qquad (71)$$

As shown in (69), the deviation of R_B from the standard model value r_B is governed by the size of R_1 and R_2.

Therefore, we start our analysis by discussing the dominant gluino contributions to R_1 and R_2. We choose the input parameters $\widetilde{m} = 250$ GeV, $x = 1$, and we obtain

$$R_1 = 0.15 \left(\delta^d_{LL}\right)_{23} (\delta^u_{LL})_{12} - 0.17 \left(\delta^d_{RR}\right)_{23} (\delta^u_{LL})_{12} + 0.18 \left(\delta^d_{RL}\right)_{23} (\delta^u_{LR})_{12} - \{L \leftrightarrow R\} \quad (72)$$

and

$$R_2 = -0.01 \left(\delta^d_{LL}\right)_{23} (\delta^u_{LL})_{21} - 0.015 \left(\delta^d_{RR}\right)_{23} (\delta^u_{LL})_{21} + 0.03 \left(\delta^d_{RL}\right)_{23} (\delta^u_{LR})_{21} - \{L \leftrightarrow R\}\,. \quad (73)$$

Using the fact that the mass insertion is less than or equal to one, we find that $R_2 \ll 1$, i.e. $A^{\rm SUSY}(B^- \to D^0 K^-) \ll A^{\rm SM}(B^- \to D^0 K)$. It is worth mentioning that $\left(\delta^d_{AB}\right)_{23}$ is constrained by the experimental results for $B \to X_s \gamma$ decay. These constraints are very weak on the LL and RR mass insertions, and they can be of order one. However, they impose stringent upper bounds on the LR and RL mass insertions, namely $|(\delta^d_{LR(RL)})_{23}| \lesssim 1.610^{-2}$ [16]. Concerning the $(\delta^u_{AB})_{12}$, the important constraints on these mass insertions are due to the D^0–$\bar{D}^0$ mixing. Applying these constraints one finds that R_1 is also quite small and the SM gives the dominant contribution. Therefore, there will be no chance to modify the results obtained in Fig. 2. However, as advocated in Sect. 4, these constraints can be relaxed, if one allows for simultaneous contributions from more than one mass insertion, which is the case in any realistic model. In this case, there may be cancellation between different contributions which reduces the SUSY contribution to D^0–$\bar{D}^0$ mixing without severely constraining the mass insertion. If we adopt this scenario and assume, for instance, that $\left(\delta^d_{LL}\right)_{23} \simeq -\left(\delta^d_{RR}\right)_{23}$ and $(\delta^u_{LL})_{12} \simeq -(\delta^u_{RR})_{12}$, then one can easily see that $R_1 \simeq \mathcal{O}(0.6)$ and the phase ϕ_1 is given by $\arg\left[\left(\delta^d_{LL}\right)_{23} + (\delta^u_{LL})_{12}\right]$.

In this case, one can easily observe that different combinations of (γ, ϕ_1) can lead to values for the $A_\pm$ within the experimental range. Therefore, the supersymmetric CP violating phases may affect the extraction of the angle γ. As an example, let us consider the case where R_B is enhanced from 0.05 (SM value) to 0.1 and the phase ϕ_B is given by $70°$, which can be obtained by $\gamma \sim \pi/3$ and $\phi_1 \sim \pi/2$ or $\gamma \sim \pi/2$ and $\phi_1 \sim \pi/3$. In this case, one finds that

$$R_+ \simeq 1.1\,, \quad R_- \simeq 0.94\,, \quad A_+ \simeq -A_- \simeq 0.2\,. \quad (74)$$

Therefore, we can conclude that the SUSY contributions to $B^- \to DK^-$ imply that A_+ and A_- are within their 1σ experimental range simultaneously, unlike the SM results.

Finally, it is important to mention that in this scenario it is a challenge to find a realistic SUSY model that accommodates these results and satisfies all other constraints. Also the observation of A_+ indicates that the ratio of the amplitudes for the processes $B^- \to \bar{D}^0 K^-$ and $B^- \to D^0 K^-$ is larger than 0.1, which is rather difficult to obtain in the SM, so it may be a hint to a new physics effect.

5.2 $R_\pm$ and $A_\pm$ with large D^0–$\bar{D}^0$ mixing

In the previous analysis, we have ignored the effect of the D–$\bar{D}^0$ mixing. Now we consider this effect and define the time dependent meson state $|D_1\rangle \equiv |D^0(t)\rangle$ and $|D_2\rangle \equiv |\bar{D}^0(t)\rangle$ by

$$|D_1\rangle = g_+(t)\left|D^0\right\rangle + \frac{q}{p} g_-(t)\left|\bar{D}^0\right\rangle\,, \quad (75)$$

$$|D_2\rangle = g_+(t)\left|\bar{D}^0\right\rangle + \frac{p}{q} g_-(t)\left|D^0\right\rangle\,, \quad (76)$$

where q/p is defined, as in Sect. 4, by

$$\frac{q}{p} = \sqrt{\frac{\mathcal{M}^*_{12}}{\mathcal{M}_{12}}} = \mathrm{e}^{-2\mathrm{i}\theta_c}\,. \quad (77)$$

As shown in (63), the phase θ_c is of order one. The functions $g_\pm(t)$ are given by [27]

$$g_\pm = \frac{1}{2}\left(\mathrm{e}^{-\mu_1 t} \pm \mathrm{e}^{-\mathrm{i}\mu_2 t}\right)\,, \quad (78)$$

with $\mu_i = M_{D_i} - \mathrm{i}\Gamma_{D_i}/2$. In terms of $x_D = \frac{\Delta M_D}{\Gamma}$ and $y_D = \frac{\Delta\Gamma}{2\Gamma}$, where $\Gamma = \Gamma_{D_1} + \Gamma_2$, one finds

$$g_+(t) = \mathrm{e}^{(-\mathrm{i}M_D t - \tau/2)}\left[1 + (x_D - \mathrm{i}y_D)^2 \tau^2/4 + \cdots\right]\,, \quad (79)$$

$$g_-(t) = \mathrm{e}^{(-\mathrm{i}M_D t - \tau/2)}\left[(-\mathrm{i}x_D - y_D)^2 \tau/2 + \cdots\right]\,. \quad (80)$$

Here $\tau = \Gamma t$. In this case, the decay amplitudes of $B^- \to DK^-$ are given by

$$A(B^- \to D_1 K^-) = A(B^- \to D^0 K^-) g_+(t) + A(B^- \to \bar{D}^0 K^-)\frac{q}{p} g_-(t) \quad (81)$$

and

$$A(B^- \to D_2 K^-) = A(B^- \to \bar{D}^0 K^-) g_+(t) + A(B^- \to D^0 K^-)\frac{p}{q} g_-(t)\,. \quad (82)$$

Also the decay rates are defined as [27]

$$\Gamma(B^- \to DK^-) = \int \mathrm{d}t \left|A(B^- \to DK^-)\right|^2\,. \quad (83)$$

Therefore, one finds that

$$\Gamma(B^- \to D_1 K^-) = \left|A(B \to D^0 K^-)\right|^2 \left(G_+ + R_B^2 G_- + 2R_B \mathrm{Re}\left[G_{+-} \mathrm{e}^{-\mathrm{i}(\delta_B + \phi_B - 2\theta_c)}\right]\right)\,, \quad (84)$$

where the G_i are given by

$$G_+ = \int_0^\infty |g_+(t)|^2\, \mathrm{d}t \simeq \frac{1}{\Gamma}\left(1 + \frac{y_D^2 + x_D^2}{2}\right)\,, \quad (85)$$

$$G_- = \int_0^\infty |g_-(t)|^2\, \mathrm{d}t \simeq \frac{1}{\Gamma}\left(\frac{y_D^2 + x_D^2}{2}\right)\,, \quad (86)$$

$$G_{+-} = \int_0^\infty g_+(t) g_-^*(t)\, \mathrm{d}t \simeq \frac{1}{\Gamma}\left(\frac{-y_D - \mathrm{i}x_D}{2}\right)\,. \quad (87)$$

The CP asymmetries $A_{CP_{1,2}}$ are defined by

$$A_{CP_{1,2}} = \frac{\Gamma(B^- \to D_{1,2}K^-) - \Gamma(B^+ \to \bar{D}_{1,2}K^+)}{\Gamma(B^- \to D_{1,2}K^-) + \Gamma(B^+ \to \bar{D}_{1,2}K^+)}\,. \tag{88}$$

Thus one can easily prove that

$$A_{CP_1} = \frac{R_B \begin{bmatrix} y_D \sin\delta_B \sin(\phi_B - 2\theta_c) \\ -x_D \sin\delta_D \cos(\phi_B - 2\theta_c) \end{bmatrix}}{G'_+ + R_B^2 G'_- - R_B \begin{bmatrix} y_D \cos\delta_B \cos(\phi_B - 2\theta_c) \\ +x_D \cos\delta_B \sin(\phi_B - 2\theta_c) \end{bmatrix}}\,, \tag{89}$$

while

$$A_{CP_2} = \frac{R_B \begin{bmatrix} y_D \sin\delta_B \sin(\phi_B - 2\theta_c) \\ +x_D \sin\delta_D \cos(\phi_B - 2\theta_c) \end{bmatrix}}{R_B^2 G'_+ + G'_- - R_B \begin{bmatrix} y_D \cos\delta_B \cos(\phi_B - 2\theta_c) \\ -x_D \cos\delta_B \sin(\phi_B - 2\theta_c) \end{bmatrix}}\,. \tag{90}$$

Here $G'_{+,-} = \Gamma G_{+,-} = \left(1 + \frac{y_D^2 - x_D^2}{2}, \frac{y_D^2 + x_D^2}{2}\right)$, respectively. The parameters x_D and y_D are subjected to stringent experimental bounds in case of $\theta_c = 0$, [28]: $x_D^2 + y_D^2 \leq (6.7 \times 10^{-2})^2$. For non-vanishing θ_c, this bound is no longer valid. However, it is believed that in general $x_D \sim y_D \sim 10^{-2}$. In this case, it is clear that $G'_+ \simeq 1$ and $G'_- \simeq 10^{-4}$, which imply that

$$A_{CP_1} \simeq 10^{-2} \times R_B \simeq \mathcal{O}(10^{-3}) \tag{91}$$

and

$$A_{CP_2} \simeq \frac{10^{-2}}{R_B} \simeq \mathcal{O}(0.1)\,. \tag{92}$$

From these results, it is remarkable that the effect of D^0–$\bar{D}^0$ mixing breaks the usual relation, $A_+ \simeq -A_-$, between the CP asymmetries $A_{CP_1} \equiv A_-$ and $A_{CP_2} \equiv A_+$, which is satisfied in the SM and SUSY models with negligible D^0–$\bar{D}^0$ mixing, as we have emphasized in the previous sections. As an example, to show how natural it is to obtain in this case CP asymmetries of the order of the central vales of the experimental results in (1), let us consider $R_B \simeq 0.15$, $x_D \simeq 3 \times 10^{-2}$, $y \simeq 5 \times 10^{-2}$, $\delta_B \sim \pi$ and $\phi_B \sim \theta_c \simeq \pi/4$. One can easily find that

$$A_{CP_1} \simeq 0.002\,, \quad A_{CP_2} \simeq 0.3\,. \tag{93}$$

It is interesting to note that these values of the CP asymmetries depend on the CP violating SM phase γ and the SUSY phase in the $b \to u$ transition ϕ_1, which contribute together to ϕ_B as in (69), in addition to the D^0–$\bar{D}^0$ mixing phase θ_c. Therefore, the determination of the angle γ relies on the new SUSY phases ϕ_1 and θ_c. This confirms the fact the our determination of the SM angle might be influenced by a new physics effect.

6 Conclusions

In this paper we have studied supersymmetric contributions to the $B^- \to D^0 K^-$ and $B^- \to \bar{D}^0 K^-$ processes. We have shown that, in the SM, the branching ratios $R_{CP_\pm}$ of these processes are within the experimental range. However, the CP asymmetry A_{CP_+} is below its 1σ experimental lower bound, and the value of A_{CP_-} is always negative. We have performed a model independent analysis of the gluino and chargino contributions to the $b \to u$ and $b \to c$ transitions. We have used the mass insertion approximation method to provide analytical expressions for all the relevant Wilson coefficients.

The D^0–$\bar{D}^0$ mixing experimental limits imply strong constraints on the mass insertions $(\delta^u_{AB})_{12}$ which affect the dominant gluino contribution to $B^- \to DK^-$. We have revised these constraints and took them into account. We showed that, in the case of negligible D^0–$\bar{D}^0$ mixing, it is possible to overcome these constraint and enhance the SUSY results for the CP asymmetries in $B^- \to DK^-$ if one consider simultaneous contributions from more than one mass insertion. In this case, the A_{CP_+} becomes within 1σ experimental range. However, with large D^0–$\bar{D}^0$ mixing, one finds a significant deviation between the two asymmetries and it becomes natural to have them of the order of the central values of their experimental measurements.

In general, we have emphasized that SUSY CP violating phases may contribute significantly to the CP asymmetries in $B^- \to DK^-$, and therefore they may affect our determination of the angle γ in the unitary triangle of the CKM mixing matrix.

Acknowledgements. I would like to thank G. Fasiel and E. Gabrielli for useful discussions.

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